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Reg. No. : .....

Code No. : 20378 E **Sub. Code : CMMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

First Semester

Mathematics – Core

CALCULUS AND CLASSICAL ALGEBRA

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The radius of curvature for the curve $y = e^x$ at the point where it crosses the y-axis is _____.

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) 2 (d) $2\sqrt{2}$

2. If $x = f(t)$ and $y = g(t)$, then the radius of curvature is _____.

(a) $\frac{(x_1^2 + y_1^2)^{3/2}}{x_1 y_1 + x_2 y_1}$ (b) $\frac{(x_1^2 + y_1^2)^{3/2}}{x_2 y_2 + x_1 y_1}$

(c) $\frac{(x_1^2 - y_1^2)^{3/2}}{x_1 y_2 - x_2 y_1}$ (d) None of these

3. In Polar co-ordinates $\frac{\partial(x, y)}{\partial(r, \theta)} =$ _____.

(a) x (b) θ

(c) r (d) $r\theta$

4. $\int_0^1 \int_0^1 xy \, dx \, dy =$ _____.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) None of these

5. $\left|\overline{\left(\frac{1}{2}\right)}\right| =$ _____

(a) $\frac{\sqrt{\pi}}{2}$ (b) $\sqrt{\pi}$

(c) π (d) $\frac{\sqrt{\pi}}{3}$

6. Transformations of Beta function $\beta(m,n)$ = _____

(a) $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ (b) $\int_1^0 x^{m-1}(1-x)^{n-1}dx$

(c) $\int_0^\infty x^{m-1}(1-x)^{n-1}dx$ (d) None of these

7. The n^{th} degree equation $f(x) = 0$ can't have more than _____ roots.

(a) 4 (b) 6

(c) 7 (d) n

8. If α, β, γ are the roots of the equation $x^3 - 2x^2 + 5x - 7 = 0$ then $\alpha\beta + \beta\gamma + \gamma\alpha$ = _____.

(a) 2 (b) 5

(c) –5 (d) 7

9. By removing the fractional co-efficients from $x^3 - \frac{1}{4}x^2 + \frac{1}{3}x - 1 = 0$ multiply the roots by _____.

(a) 4 (b) 3

(c) –3 (d) 12

10. The equation $z^3 - 3z + 1 = 0$ has a real root between _____.

(a) 1,2 (b) 2,3

(c) 3,4 (d) 4,5

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the radius of curvature of the cardioid $r = a(1 - \cos\theta)$.

Or

(b) For the curve $x^3 + y^3 = 3axy$, show that the radius of curvature is $\frac{3\sqrt{2}\,a}{16}$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.

12. (a) If $x + y + z = u$, $y + z = uv$, $z = uvw$ prove $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$.

Or

(b) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$.

13. (a) Prove that $\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^5\theta \, d\theta = \frac{1}{120}$.

Or

(b) Simplify $\int_0^{\frac{\pi}{2}} \sin^{10}\theta \, d\theta$.

14. (a) If α, β, γ are the roots of the equation $x^3 + px^2 + r = 0$ find the value of $\alpha^3 + \beta^3 + \gamma^3$.

Or

(b) Frame an equation with rational co-efficient, one of whose roots is $\sqrt{5} + \sqrt{2}$.

15. (a) Solve $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$

Or

(b) Transform the equation $x^4 + x^3 - 3x^2 + 2x - 4 = 0$ whose roots are each diminished by 2.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find 'P' at the point 't' of the curve $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$

Or

(b) Find the equation of the evolute of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

17. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx \, dy \, dz$.

Or

(b) Change the order of integration and evaluate $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xy \, dx \, dy$.

18. (a) Show that $\beta(m,n) = \frac{\lceil m \lceil n}{\lceil (m+n)}$

Or

(b) Express $\int_0^1 x^m(1-x^n)^p dx$ interms of Gamma function and evaluate $\int_0^1 x^5(1-x^3)^{10} dx$.

19. (a) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ are in G.P.

Or

(b) Find correct to two place of decimals the root of the equation $x^4 - 3x + 1$ that lies between 1 and 2 by Newton method.

20. (a) Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$

Or

(b) Discuss the nature of the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$.